ตัวแผ่กระจายของทฤษฎีสนามควอนตัมในสภาวะไม่สมดุลที่มีสมบัติกลุ่มเชิงตั้งฉาก

PROPAGATORS OF THE NONEQUILIBRIUM O(N) QUANTUM FIELD THEORY

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ทฤษฎีสนามควอนตัมในสภาวะไม่สมดุลดำเนินการกับสถานการณ์ที่ปริมาณทางฟิสิกส์ขึ้นกับเวลา ในงานนี้ปริพันธ์ตามวิถีปิดและรูปนัยนิยมที่ลดทอนไม่ได้ด้วยสองอนุภาคซึ่งถูกใช้ในการสร้างรูปแบบของทฤษฎีสนามควอนตัมในสภาวะไม่สมดุลได้ถูกทบทวน ซึ่งกระบวนการทั้งสองนี้ถูกนำไปใช้ในการอนุพัทธ์สมการพลวัตที่สอดคล้องกับสนามเฉลี่ยและตัวแผ่กระจายในทฤษฎีของสนามไร้ทิศทางที่มีสมบัติกลุ่มเชิงตั้งฉากในสภาวะไม่สมดุล โดยมีอันตรกิริยาเป็นแบบกำลังสี่ โดยสมการพลวัตที่ได้เป็นสมการคู่ควบที่สอดคล้องกับสนามเฉลี่ยและตัวแผ่กระจาย

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ABSTRACT

Nonequilibrium quantum field theory deals with the situations in which the physical quantities are time-dependent. In this work, the closed-time path integral and the 2-paricle irreducible formalism used to formulate the nonequilibrium quantum field theory are reviewed. These methods are then used to derive the dynamical equations satisfied by the mean fields and the propagators of the nonequilibrium O(N) scalar field theory with quartic self-interactions. The dynamical equations obtained are coupled equations satisfied by the meand fields and the propagators.

Keywords: nonequilibrium quantum field theory, O(N) model, 2-particle irreducible formalism, closed-time path integral.

Introduction

Nonequilibrium quantum field theory has been of interest among theoretical physicists during the past several years, due to the fact that many important phenomena in particle physics and cosmology occur in the nonequilibrium situation. Unlike the conventional quantum field theory, nonequilibrium quantum field theory employs the closed-timed path integral technique and the 2-particle irreducible formalism, which may be unfamiliar to many physicists. The closed-time path integral method (CTP) was first developed by Schwinger and later on generalized by Keldysh, and the unified approach applicable to both equilibrium and nonequilibrium systems was reviewed by Chou, Su, Hao and Yu (1984). Along with the CTP technique, the 2-particle irreducible formalism (2PI), which was introduced by Cornwall, Jackiw and Tomboulis (1974), is a necessary tool for nonequilibrium quantum field theory. By using the CTP and 2PI, nonequilibrium quantum field theory allows us to treat the propagators as the independent quantities, which is different from the conventional quantum field theory. In this note, we will, therefore, begin with a brief review of the concepts of the closed-time path integration and the 2-particle irreducible formalism, and then go on to analyze the $O(N)$ scalar field theory with quartic self-interactions in the nonequilibrium setting. Since the propagators are now independent variables, there must exist dynamical equations satisfied by propagators. Our aim is to obtain these dynamical equations along with the equations satisfied by the mean fields.

Objective

The purpose of this work was to formulate nonequilibrium $O(N)$ scalar field theory with quartic self-interactions and to find the dynamical equations satisfied by the propagators in this model.

Closed-time path integral

Before discussing the closed-time path integral formalism, we first recall that an important object in the conventional quantum field theory is the generating functional (or the vacuum-to-vacuum transition amplitude) in the presence of an external source J, defined by $Z[J] = \langle 0(+\infty) | U_J(+\infty,-\infty) | 0(-\infty) \rangle$, where $U_J(+\infty,-\infty)$ is the time-evolution operator linking the vacuum states in the infinite past and in the infinite future. For a scalar field theory, $U_J(+\infty,-\infty) = T\left(\exp i(H + \Phi J)/\hbar\right)$, where H and Φ are the Hamiltonian and the scalar field, respectively, and T is the time-ordering operator. In the above expression, an integral over d^4x of the exponent is understood without explicitly writing it; we shall use this convention from now on. It is important to note that the vacuum states in the definition of $U_J(+\infty,-\infty)$ are the ground states at different time, and it is possible that these ground states are different when the system under consideration undergoes a nonequilibrium phase. Using the generating functional, the effective action $\Gamma[\phi]$ is defined as the Legendre transform of $W[J] \equiv -i\hbar \ln(Z[J])$, i.e. $\Gamma[\phi] = W[J] - \phi J$, where $\phi(x) = \delta W[J]/\delta J(x)$. Using this definition of $\phi(x)$, one can see that if the vacua in the asymptotic past and in the far future are equivalent in the sense that the vacuum state in the asymptotic past evolves uniquely into the vacuum state in the far future, $\phi(x)$ is just the ground-state value of the field operator $\Phi(x)$ at time $t = x^0$, and the value of $\phi(x)$ can be obtained by solving the equation $\delta\Gamma[\phi]/\delta\phi = 0$. However, in the nonequilibrium situation in which things change with time, all we know are the initial conditions of the system, and so we cannot be sure if the vacuum state in the infinite past will evolve uniquely into the vacuum state in the far future. Having no idea of what the vacuum state in the far future looks like, it is impossible to calculate the generating functional defined above, and so the above formalism fails. To overcome this difficulty, one introduces the closed-time path integral (CTP) formalism, with a new generating functional defined by (Jordan, 1986 : 445)

$$Z[J_1, J_2] = \langle 0(-\infty) | U_{J_2}(-\infty,+\infty) U_{J_1}(+\infty,-\infty) | 0(-\infty) \rangle \quad (1)$$

where $U_{J_1}(+\infty,-\infty)$ was defined above, and $U_{J_2}(-\infty,+\infty)$ is defined similarly but with the anti-temporal ordering operator instead of the time-ordering one. The interpretation is as follows. Starting with the ground state $|0(-\infty)\rangle$ in the asymptotic past, we evolve it using $U_{J_1}(+\infty,-\infty)$ (with the source J_1) into the infinite future, and then bring it back to the infinite past using $U_{J_2}(-\infty,+\infty)$ (with the source J_2), where it becomes the initial ground state $|0(-\infty)\rangle$ again. We thus see that this formalism does not require any knowledge about things in the future; all we need to know are the initial conditions at $t=-\infty$. If we let $W[J_1, J_2] = -i\hbar \ln Z[J_1, J_2]$, it is easy to see that $\phi(x) \equiv (\delta W/\delta J_1(x))|_{J_1=J_2}$ is the vacuum expectation value of Φ at time $t = x^0$, where the vacuum state at time x^0 is the one evolving from $|0(-\infty)\rangle$ by $U_{J_1}(x^0, -\infty)$. It is important to note that the condition $J_1 = J_2$ in the definition of $\phi(x)$ results in the disappearance of the contributions from things at time $t > x^0$ in the calculation of $\phi(x)$, so that the causality is automatic.

The path integral representation of Eq. (1) can be formulated using the following trick. We first enlarge the size of the time dimension by a factor of two, where t runs from $-\infty$ to $+\infty$ in the first half, and from $+\infty$ back to $-\infty$ in the second half. (Thus the time dimension has the structure of a closed loop, hence the name "closed-time path integral.") If we denote the scalar field by Φ_1 (with the source J_1) in the first half of the time dimension, and by Φ_2 (with the source J_2) in the second half, it is clear that the path integral representation of Eq. (1) takes the form (Jordan, 1986 : 446)

$$Z[J_1, J_2,] = \int D\Phi_1 D\Phi_2 \exp\frac{i}{\hbar}\{(S[\Phi_1] + J_1\Phi_1) - (S^*[\Phi_2] + J_2\Phi_2)\} \qquad (2)$$

subject to the conditions that $\Phi_1(+\infty) = \Phi_2(+\infty)$ and $J_1(+\infty) = J_2(+\infty)$. Note that the minus sign in front of the Φ_2 part of the exponent came from the fact that Φ_2 propagates backward in time while the integral over time is defined in the forward time direction. Even though we formally have only one scalar field propagating along two time branches (one from $t=-\infty$ to $+\infty$ and the other from $t=+\infty$ to $-\infty$), $Z[J_1, J_2]$ has the mathematical structure of a functional integral over two scalar fields, so we can evaluate it in the conventional way. Introducing the metric $c_{ab} = c^{ab} = \mathrm{diag}(1,-1)$ ($a, b = 1,2$), we can use it to raise and lower the indices according to the rule $J^a = c^{ab} J_b$ and $\Phi^a = c^{ab}\Phi_b$. Using this metric, the generating functional can be written in the more compact form as

$$Z[J^a] = \int D\phi_a \exp\frac{i}{\hbar}(S[\Phi_a] + J^a\Phi_a), \tag{3}$$

where $S[\Phi_a] \equiv S[\Phi_1] - S^*[\Phi_2]$. The corresponding effective action is defined as a Legendre transform of $W[J^a] = -i\hbar\ln(Z[J^a])$,

$$\Gamma[\phi_a] = W[J^a] - \phi_a J^a \tag{4}$$

where $\phi_a(x) \equiv \delta W/\delta J^a(x)$. The rest of the calculation is the same as that in the conventional quantum field theory. A minute thought reveals that the vacuum expectation value of the original scalar field is $\phi(x) = \delta W/\delta J^a(x)|_{J_1=J_2=J}$, which is obtained by solving the equation $\delta\Gamma/\delta\phi_a|_{\phi_1=\phi_2=\phi} = 0$, which describes the time-evolution of $\phi(x)$ (remember that, unlike ordinary quantum field theory, $\phi(x)$ is time dependent in the nonequilibrium situation). It is worth mentioning that this time evolution should respect the causality, since $\phi(x)$ depends on things that happened only in its past, by construction.

Let us now consider the propagators of the theory. We first recall that a propagator is the vacuum expectation value of the time-ordering product of two scalar fields. In the closed-time path integral formalism, this time ordering is defined to be along the direction of the closed-time path. As there are two scalar fields, one for each time branch, it is clear that there are four types of propagators; the first two being formed by the fields on the same time branch, while the others being constructed from the fields on different time branches. In the functional method, these propagators are obtained by performing the functional differentiation on $W[J^a]$,

$$G_{ab}(x,x') = \frac{\delta W[J_1, J_2]}{\delta J^a(x')\delta J^b(x)}, \tag{5}$$

and then setting $J_1 = J_2$, with the result:

$$\left.\frac{\delta W[J_1, J_2]}{\delta J^1(x')\delta J^1(x)}\right|_{J_1=J_2=J} = \left\langle T(\Phi(x)\Phi(x'))\right\rangle = G_F(x, x') \tag{6}$$

$$\left.\frac{\delta W[J_1, J_2]}{\delta J^2(x)\delta J^1(x')}\right|_{J_1=J_2=J} = \left\langle \Phi(x)\Phi(x')\right\rangle = G_+(x, x') \tag{7}$$

$$\left.\frac{\delta W[J_1, J_2]}{\delta J^2(x')\delta J^1(x)}\right|_{J_1=J_2=J} = \left\langle \Phi(x')\Phi(x)\right\rangle = G_-(x, x') \tag{8}$$

$$\left.\frac{\delta W[J_1, J_2]}{\delta J^2(x')\delta J^2(x)}\right|_{J_1=J_2=J} = \left\langle \widetilde{T}(\Phi(x)\Phi(x'))\right\rangle = G_D(x, x') \tag{9}$$

where G_F, G_+, G_-, G_D are Feynman, positive, negative and Dyson propagators, respectively. Observe that the Dyson propagator is defined using the reverse time ordering, due to the fact that it contains only the fields on the reverse time branch (on which time runs backward). Thus, unlike the conventional quantum field theory in which only the Feynman propagator exists, there are four propagators that contribute to each internal line of a Feynman diagram. Since each interaction vertex is defined on either forward or reverse time branch, it is not hard to see that a vacuum Feynman diagram with n vertices in the conventional quantum field theory may be thought of as describing 2^n diagrams in the CTP formalism.

2-particle irreducible formalism

When dealing with the nonequilibrium problem, we are normally given an information about the probability distribution of the initial states (at $t = -\infty$), and then asked what will happen in the future given these initial conditions. It is clear that this initial probability distribution is the thing that was missing in the above formulation of the closed-time path integral, which is supposed to describe the nonequilibrium system. This remark tells us that the CTP generating functional introduced previously has to be modified in order to completely describe the nonequilibrium situation. It turns out that the correct CTP generating functional takes the form

$$Z[J_1, J_2, \rho] = Tr[U_{J_2}(-\infty,+\infty)U_{J_1}(+\infty,-\infty)\rho(-\infty)] \quad (10)$$

where $\rho(-\infty)$ is the density operator at the initial time, which contains all information about the probability distribution of initial states, and the trace is taken over all initial states. The corresponding path integral representation is found to be

$$Z[J^a, \rho] = \int D\Phi_a \exp\frac{i}{\hbar}\{(S[\Phi_a] + J^a\Phi_a)\}\langle\Phi_1(-\infty)|\rho|\Phi_2(-\infty)\rangle, \quad (11)$$

where $\langle\Phi_1(-\infty)|\rho|\Phi_2(-\infty)\rangle$ is the matrix element of the density operator with respect to the initial states. Here, the functional integral over Φ_a is subject to the constraint that each field approaches its given initial state $\Phi_a(-\infty)$ as $t \to -\infty$, and the functional integration over the initial states ($\Phi_1(-\infty)$ and $\Phi_2(-\infty)$) is understood without explicitly writing it.

Since the matrix elements of the density operator cannot be identically zero, it is tempting to express them in the exponential form as (Calzetta & Hu, 1988 : 2883)

$$\langle\Phi_1(-\infty)|\rho|\Phi_2(-\infty)\rangle = \exp\left(\frac{i}{\hbar}K[\Phi_a]\right) \quad (12)$$

where

$$K[\Phi_a] = K + K^a(x)\Phi_a(x) + \frac{1}{2}K^{ab}(x,y)\Phi_a(x)\Phi_b(y) + \ldots \quad (13)$$

In the above expression of $K[\Phi_a]$, the integration over all spacetime coordinates should be understood without having to write it, and all kernels $K, K^a, K^{ab}, \ldots$ must be non-vanishing only at $t = -\infty$, so that $K[\Phi_a]$ contains information only at the initial time. With this form of $K[\Phi_a]$, all information about the initial state of the system is thus contained in the kernels. Using the above form of the matrix elements, the generating functional now contains an infinite number of nonlocal terms, and so it is a formidable task to evaluate this functional integral. To simplify this task, one might try to retain only a small number of terms in the infinite series, but this will result in the loss of some information about the initial probability distribution.

Anyway, if we insist on doing this and keep only the lowest-order nonlocal term, the generating functional becomes

$$Z[J^a, K^{ab}] = \int D\Phi_a \exp\frac{i}{\hbar}\{S[\Phi_a] + J^a\Phi_a + \frac{1}{2}K^{ab}(x,y)\Phi_a(x)\Phi_b(y)\} \tag{14}$$

Let $W[J^a, K^{ab}] \equiv -i\hbar \ln Z[J^a, K^{ab}]$, we define

$$\frac{\delta W[J^a, K^{ab}]}{\delta J^a} = \phi_a, \tag{15}$$

$$\frac{\delta W[J^a, K^{ab}]}{\delta K^{ab}} = \frac{1}{2}(\phi_a\phi_b + \hbar G_{ab}), \tag{16}$$

and then define the effective action as a Legendre transform of $W[J_a, K_{ab}]$,

$$\Gamma[\phi_a, G_{ab}] = W[J^a, K^{ab}] - J^a\phi_a - \frac{1}{2}K^{ab}(\phi_a\phi_b + \hbar G_{ab}), \tag{17}$$

which implies

$$\frac{\delta\Gamma[\phi_a, G_{ab}]}{\delta\phi_a} = -J^a - \frac{1}{2}\phi_b(K^{ab} + K^{ba}), \tag{18}$$

$$\frac{\delta\Gamma[\phi_a, G_{ab}]}{\delta G_{ab}} = -\frac{\hbar}{2}K^{ab}. \tag{19}$$

It is easy to see that $\phi_a(x)$ is the expectation value of $\Phi_a(x)$ at time x^0. To find the meaning of G_{ab}, we first express $\Phi_a(x)$ as the sum of its expectation value $\phi_a(x)$ and the fluctuation $\varphi_a(x)$, $\Phi_a = \phi_a + \varphi_a$, so that the action becomes

$$S[\phi_a + \varphi_a] = S[\phi_a] + \frac{\delta S[\phi_a]}{\delta\phi_a}\varphi_a + \frac{1}{2}\frac{\delta^2 S[\phi_a]}{\delta\phi_a\delta\phi_b}\varphi_a\varphi_b + S_Q, \tag{20}$$

with S_Q describing the interactions among the field fluctuations, and the functional integral over Φ_a becomes an integral over φ_a. By imposing the initial conditions that $\phi_a(t = -\infty) = \Phi_a(-\infty)$, we see that φ_a vanishes at $t = -\infty$. If we also assume that φ_a also vanishes at $t = +\infty$, then we can evaluate the functional integral over φ_a in the same way as we did in ordinary quantum field theory, without having to worry about the constraints due to the initial conditions. With this field decomposition, we can evaluate the generating functional, and subsequently obtain the effective action with the result

$$\Gamma[\phi_a, G_{ab}] = S[\phi_a] + \frac{\hbar}{2} \frac{\delta^2 S}{\delta\phi_a \delta\phi_b} G_{ab} - \frac{i\hbar}{2} Tr \ln G + \Gamma_2 \tag{21}$$

where Γ_2 is the sum of 2-particle irreducible (2PI) diagrams produced by field fluctuations. To verify this property of Γ_2, we substitute the above result into Eq. (19), with the result

$$i(G^{-1})^{ab} = -K^{ab} + \frac{\delta^2 S}{\delta\phi_a \delta\phi_b} + \frac{2}{\hbar} \frac{\delta\Gamma_2}{\delta G_{ab}} \tag{22}$$

which implies that G_{ab} is the propagator of the field fluctuations φ_a if K^{ab} is set to zero. Using the fact that $(G^{-1})^{ab}$, which is equal to $\delta\Gamma_2/\delta G_{ab}$ plus a classical inverse propagator (without quantum corrections), is the sum of 1PI diagrams, we conclude that Γ_2 is the sum of 2PI diagrams with G_{ab} as the propagators, and therefore can be evaluated without much difficulty. It should be noted that the propagators in Eq. (22) are the same as the one defined in Eq. (5). $\Gamma[\phi_a, G_{ab}]$ is thus known as the 2PI effective action (Cornwall, Jackiw & Tomboulis, 1974 : 2431). Once Γ_2 has been evaluated, we can substitute the resulting effective action back into Eqs. (18) and (19), and obtain the equations for determining ϕ_a and G_{ab} by setting $J^a = K^{ab} = 0$. By setting $\phi_1 = \phi_2 = \phi$, the equations for determining the field expectation value ϕ and the propagators are obtained (Calzetta & Hu, 1988 : 2885).

Propagators of the nonequilibrium O(N) scalar field theory

We now use the formulation developed above to find the equations for determining the expectation values of the fields and the propagators of the $O(N)$ spinless model. In this model, there are N scalar fields ϕ^i ($i = 1, 2, \ldots, N$), and the action, invariant under the action of an orthogonal group $O(N)$ on the fields, with quartic self-interactions, takes the form (Ramsey & Hu, 1997 : 669)

$$S[\phi^i] = \frac{\delta_{ij}}{2}(-\partial^{\mu}\phi^i\partial_{\mu}\phi^j - m^2\phi^i\phi^j) - \frac{\lambda}{8N}(\delta_{ij}\phi^i\phi^j)^2. \quad (23)$$

The corresponding generating functional is thus

$$Z[J_i^a, K_{ij}^{ab}] = \int D\Phi_a^i \exp\{\frac{i}{\hbar}S[\Phi_a^i] + J_i^a\Phi_a^i + \frac{1}{2}K_{ij}^{ab}\Phi_a^i\Phi_b^j\}. \quad (24)$$

Note that there are two types of indices, the first one denoted by early roman alphabets ($a, b, c, \ldots$) labels the time branches while the second one denoted by late roman letters ($i, j, k, \ldots$) represents the $O(N)$ indices. The corresponding effective action then takes the form

$$\Gamma[\phi_a^i, G_{ab}^{ij}] = S[\phi_a^i] - \frac{i\hbar}{2} Tr \ln G + \frac{\hbar}{2}\frac{\delta^2 S[\phi_a^i]}{\delta\phi_a^i\delta\phi_b^j} G_{ab}^{ij} + \Gamma_2, \quad (25)$$

where Γ_2 can be obtained by summing over 2PI diagrams produced by the following interaction terms

$$S_Q[\varphi_a^i] = -\frac{\lambda}{2N}\{(c^{ab}\delta_{ij}\varphi_a^i\varphi_b^j)(c^{ab}\delta_{ij}\varphi_a^i\varphi_b^j) + \frac{1}{4}(c^{ab}\delta_{ij}\varphi_a^i\varphi_b^j)^2\}. \quad (26)$$

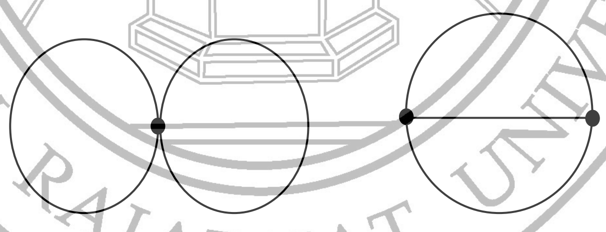

Figure 1: The double-bubble and sunset diagrams.

The lowest-order contribution to Γ_2 consists of two-loop diagrams in the forms of the double-bubble and sunset diagrams as shown in Fig. 1. To the lowest order, the result is (Ramsey & Hu, 1997 : 672)

$$\Gamma_2 = \frac{\hbar^2\lambda}{4N}\{-\frac{1}{2}c^{abcd}(G_{ab}^{ij}(x,x)G_{cd}^{kl}(x,x) + 2G_{ab}^{ik}(x,x)G_{cd}^{jl}(x,x))\delta_{ij}\delta_{kl}$$

$$+\frac{i\lambda}{N}c^{abcd}c^{a'b'c'd'}\phi_a^i\phi_{a'}^{i'}(G_{bb'}^{ii'}(x,x')G_{cc'}^{jj'}(x,x')G_{dd'}^{kk'}(x,x')$$
$$+2G_{bd'}^{ij'}(x,x')G_{cc'}^{jk'}(x,x')G_{db'}^{ki'}(x,x'))\delta_{jk}\delta_{j'k'}\} \quad (27)$$

where $c_{abcd}=c^{abcd}$ is equal to 1 for $a=b=c=d=1$, and is equal to -1 for $a=b=c=d=2$. The first term came from the double-bubble diagrams, while the sunset diagrams gave the second term containing ϕ's. Using the above result, we obtain the equations for determining the propagators

$$i(G^{-1})_{ij}^{ab}(x,x')=\frac{\delta^2 S[\phi_a^i(x)]}{\delta\phi_b^j(x')\delta\phi_a^i(x)}-\frac{\hbar\lambda}{2N}c^{abcd}\delta^4(x-x')\{\delta_{ij}\delta_{kl}G_{cd}^{kl}+2\delta_{ki}\delta_{lj}G_{cd}^{kl}\}$$
$$+\frac{i\hbar\lambda^2}{2N^2}c^{acde}c^{bc'd'e'}\delta_{kk'}\delta_{ll'}\delta_{im}\delta_{jn}\{\phi_c^m\phi_{c'}^n G_{dd'}^{kl}G_{ee'}^{k'l'}+2\phi_c^k\phi_{c'}^l G_{dd'}^{k'l'}G_{ee'}^{mn}$$
$$+2\phi_c^m\phi_{c'}^k G_{dd'}^{ln}G_{ee'}^{l'k'}+2\phi_c^k\phi_{c'}^l G_{dd'}^{k'n}G_{ee'}^{ml'}+2\phi_c^k\phi_{c'}^n G_{dd'}^{k'l}G_{ee'}^{ml'}\} \quad (28)$$

and the ones for determining ϕ_a^i

$$(c^{cb}(\Omega+m^2)+c^{abcd}\frac{\lambda}{2N}\phi_a^i\phi_d^j\delta_{ij})\phi_b^m+\frac{\hbar\lambda}{2N}c^{abcd}\{\delta_{ij}\phi_d^m G_{ab}^{ij}+\delta_{jl}\delta_i^m\phi_d^l(G_{ab}^{ij}+G_{ab}^{ji})\}$$
$$-\int d^4x'\frac{i\hbar^2\lambda^2}{4N^2}\Sigma^{cm}(x,x')=0 \quad (29)$$

valid at the two-loop level. The nonlocal term $\Sigma^{cm}(x,x')$ which appears above has the explicit form

$$\Sigma^{cm}(x,x')=c^{cabd}c^{c'a'b'd'}\phi_{c'}^i\{G_{aa'}^{mi'}G_{bb'}^{jj'}G_{dd'}^{kk'}+2G_{ad'}^{mj'}G_{bb'}^{jk'}G_{a'd}^{ki'}$$
$$+G_{a'a}^{i'm}G_{b'b}^{jj'}G_{d'd}^{kk'}+2G_{a'd}^{i'j'}G_{b'b}^{kk'}G_{d'a}^{jm}\}\delta_{jk}\delta_{j'k'}\delta_{ii'}. \quad (30)$$

By setting $\phi_1^i=\phi_2^i=\phi^i$ in the above equations, we obtain the set of equations for determining the expectation values of the fields ϕ^i and the propagators of the nonequilibrium $O(N)$ model, which can be solved once the initial conditions have been specified.

As mentioned earlier, causality is very important and so the dynamical equations for the mean fields ϕ_a^i should respect it. This means that the nonlocal term

$\Sigma^{cm}(x,x')$, generally referred to as the memory function, must vanish unless $x^0 > x'^0$. To show that this is indeed the case, we define the retarded, advanced and correlation Green's functions by $G_R = i(G_F - G_-)$, $G_A = -i(G_D - G_+)$ and $G_C = (G_- + G_+)$, respectively. By setting $\phi_1^i = \phi_2^i = \phi^i$ and writing all propagators in Eq. (30) in terms of G_R, G_A and G_C, it can be verified that all terms on the right hand side of Eq. (30) are proportional to the retarded Green's function, which means that the nonlocal term $\Sigma^{cm}(x,x')$ respects the causality. Thus causality is respected by all equations which determine the field expectation values and the propagators as expected.

Conclusions and discussion

The CTP technique and the 2PI formalism were used to formulate the nonequilibrium quantum field theory. The propagators are treated as independent variables and their dynamical equations can be obtained from the effective action. A specific case of the nonequilibrium $O(N)$ scalar field theory with quartic self-interactions was analyzed, and the equations which describe the dynamics of the theory were obtained. Unfortunately, these dynamical equations determining the mean-field and the propagators are coupled to each other. Due to their complicated forms, solving these equations is a formidable task. Despite this difficulty, we found that these equations respect the causality as they should.

Our suggestion is that one may try to solve these dynamical equations by numerical methods. But since the propagators are normally complex functions, a convenient way is to split the propagators into their real and imaginary parts before solving the dynamical equations numerically.

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