Hybrid Algorithm to Newton Raphson method and Bisection method

Chalermwut Comemuang[1], Pairat Janngam[2,*]

[1]Department of Mathematics, Faculty of Science, Buriram Rajabhat University, Buriram, Thailand.

[2] Department of Mathematics, Faculty of Education, Buriram Rajabhat University, Buriram, Thailand.

Abstract: The purpose of this research propose is to hybrid algorithm to Newton-Raphson method and Bisection method to compute roots of nonlinear equations. Numerical experiments for various tests nonlinear equations confirm performance for the bracketing method or open method to be compared with.

1. INTRODUCTION

Computing roots of nonlinear equations by iteration is a significant topic for numerical analysis. Because of the complicated problems of science, computer science and engineering, it cannot find the exact answer by the methods for solving equation. So, iteration is an alternative choice to find the answer or it could be called the step of finding root of function x which $f(x) = 0$ it is one of the well-known iteration method for finding answers called Newton-Raphson method and Bisection method. The derivative of function must be used for calculating the answer of every conducting iteration and has square convergence which is said to be the effective way to find the answer quickly. Then, there are a lot of researchers improving Newton-Raphson Method to be more convergent and one of them was Homeier[1] who presented the adjusted Newton-Raphson Method to have cubic convergence. Changbum and Beny[2] had improved fourth-order convergence. G. Mahesh[3] proposed a new method to compute a non-zero real root of the

*Corresponding author

E-mail address: pairat.jn@bru.ac.th

transcendental equations. Besides, there is a bisection method which is marginal specification with the notion of finding answers by using the intermediate value theorem to create closed interval containing the answers. The convergent sequence is linear, and it can certainly find the answer. However, this method could be used to find answer quite slowly, so Sabharwal[4] had improved bisection and false position method for finding the answers more swiftly. Kouider[5] presented a method of improved bisection and false position for many square root values through R program. Ali et al.[6] had improved bisection method by divided n as sub range. Tanakan[7] had improved bisection through the notion of Secant Method to find the answer rapidly. Neamvonk[8] has been a new method for solving nonlinear equations by using nonlinear regression. Hafiz [9] proposed an improved method called BRFC, by the combination of the Bisection, Regula Falsi and parabolic interpolation.

However, the method of marginal specification, square roots converged quite slowly, but Newton-Raphson method cannot find the certain answers depending on choosing appropriate beginning point, so Altaee, Hoomod and Hussein[10] presented hybrid algorithm among Newton-Raphson method. Kim, Noh, Oh and Park[11] had created the step of hybrid which used bisection and Newton-Raphson method to improve hybrid method of Altaee et al.

Therefore, this study revealed that the improvement of hybrid among Newton-Raphson method and bisection are more effective for convergence than interval or open approach.

2. MAIN RESULTS

Let f be a continuous function and defined on [a, b] which $f(a) \cdot f(b) < 0$. Firstly, we set

$$a1 = a - \frac{f(a)}{f'(a)}$$
$$b1 = b - \frac{f(b)}{f'(b)}$$

we have two main cases:

case 1: If $a1$ or $b1 \notin [a,b]$, By the bisection method, we have $c = \frac{a+b}{2}$ .Next, we consider a

new subinterval $(a*, b*)$ by

$$(a*, b*) = \begin{cases} a* = c, b* = b & \text{,if } f(a) \cdot f(c) < 0 \\ a* = a, b* = c & \text{,if } f(a) \cdot f(c) > 0 \end{cases}$$

case 2: If $a1$ or $b1 \in [a, b]$

case 2.1: If $f(a1) \cdot f(b1) < 0$ then $c = \dfrac{a1 + b1}{2}$. Next, we consider a new subinterval

$(a*, b*)$ by

$$(a*, b*) = \begin{cases} a* = a1, b* = b & \text{,if } f(a1) \cdot f(c) < 0 \\ a* = c, b* = b1 & \text{,if } f(a1) \cdot f(c) > 0 \end{cases}$$

case 2.2: If $f(a1) \cdot f(b1) > 0$, we set $a* = a1, b* = b1$ and

$$c = \begin{cases} a1 & \text{,if } |f(a1)| < |f(b1)| \\ b1 & \text{,if } |f(a1)| > |f(b1)| \end{cases}$$

Finally, we choose the new subinterval for the next iteration as follows $a = a*$ and $b = b*$

The process is continued until the interval is sufficiently small or the approximate solution is sufficiently close to the exact solution.

Therefore, we can state the algorithm for finding a solution of nonlinear equation $f(x) = 0$ on an interval $[a, b]$ as follows:

2.1. The algorithm

Hybrid Algorithm (HA)

Step 1 : Get $f(x), f'(x), [a, b]$ (interval)

Step 2 : $a1 = a - \dfrac{f(a)}{f'(a)}, b1 = b - \dfrac{f(b)}{f'(b)}$

Step 3 : $a1$ or $b1 \notin [a, b]$, compute $c = \dfrac{a + b}{2}$ and

$$(a^*, b^*) = \begin{cases} a^* = c, b^* = b & \text{,if } f(a) \cdot f(c) < 0 \\ a^* = a, b^* = c & \text{,if } f(a) \cdot f(c) > 0 \end{cases}$$

go to Step 6.

Step 4 : $f(a1) \cdot f(b1) < 0$ compute $c = \dfrac{a1 + b1}{2}$

$$(a^*, b^*) = \begin{cases} a^* = a1, b^* = c & \text{,if } f(a1) \cdot f(c) < 0 \\ a^* = c, b^* = b1 & \text{,if } f(a1) \cdot f(c) > 0 \end{cases}$$

go to Step 6.

Step 5 : $f(a1) \cdot f(b1) > 0$, we set $a^* = a1, b^* = b1$ and

$$c = \begin{cases} a1 & \text{,if } |f(a1)| < |f(b1)| \\ b1 & \text{,if } |f(a1)| > |f(b1)| \end{cases}$$

Step 6 : If $f(c) < \varepsilon$ then the zero is c. Stop.

Step 7 : Set $a = a^*$ and $b = b^*$ and to Step 2.

3. Numerical examples

In this section, we compare our proposed methods Bisection Algorithm(BS)[12], Regula Falsi Algorithm(RF)[13], Improved Regula Falsi Algorithm(IRF)[13], Bisection Newton-like Algorithm(AC)[14], Newton-Raphson Algorithm(NR), Proposed Method(PP) and Hybrid Algorithm(HA) we tested the algorithms with the specific examples on Scilab (version 6.1.0) program. For the accuracy, we use tolerance error(TOL) less then $\varepsilon = 1.0 \times 10^{-10}$ and also the following criteria is used for estimating the zero $|f(x)| < \varepsilon$

$f(x)$	Methods	Interval	No. of iteration	$|f(x)|$
$f_1(x) = x^2 - (1-x)^5$	BS	[0.1, 1]	33	3.29e-11
	RF	[0.1, 1]	20	4.83e-11
	IRF	[0.1, 1]	5	1.74e-14
	AC	[0.1, 1]	6	3.22e-13
	NR	0.1	5	0.00e+00

Hybrid Algorithm method

$f(x)$	Methods	Interval	No. of iteration	$\left|f(x)\right|$
	PP	0.1	4	6.93e-17
	HA	[0.1, 1]	4	8.78e-12
$f_2(x) = \cos(x) - x^3$	BS	[0.1, 1]	34	4.65e-11
	RF	[0.1, 1]	12	3.59e-11
	IRF	[0.1, 1]	5	1.82e-11
	AC	[0.1, 1]	34	4.65e-11
	NR	0.1	10	2.95e-12
	PP	0.1	19	1.17e-12
	HA	[0.1, 1]	4	5.22e-15
$f_3(x) = xe^x - 1$	BS	[-1, 1]	34	2.84e-11
	RF	[-1, 1]	22	3.37e-11
	IRF	[-1, 1]	6	3.33e-16
	AC	[-1, 1]	8	2.27e-11
	NR	-1	2	0.00e+00
	PP	-1	1	0.00e+00
	HA	[-1, 1]	5	2.22e-16
$f_4(x) = x^2 - e^x - 3x + 2$	BS	[-2, 2]	36	4.95e-11
	RF	[-2, 2]	19	9.11e-11
	IRF	[-2, 2]	4	1.09e-13
	AC	[-2, 2]	36	4.95e-11
	NR	-2	5	0.00e+00
	PP	-2	∞	∞
	HA	[-2, 2]	3	1.15e-12
$f_5(x) = \sin(x) - \dfrac{x}{2}$	BS	[-3, 5]	35	6.36e-11
	RF	[-3, 5]	24	4.32e-11
	IRF	[-3, 5]	11	1.65e-11
	AC	[-3, 5]	35	6.36e-11
	NR	-3	5	0.00e+00
	PP	-3	5	1.32e-13
	HA	[-3, 5]	5	0.00e+00
$f_6(x) = x^3$	BS	[-0.5, 1/3]	9	3.44e-11
	RF	[-0.5, 1/3]	579117	1.00e-10
	IRF	[-0.5, 1/3]	463465	1.00e-10
	AC	[-0.5, 1/3]	7	7.84e-11
	NR	-0.5	18	3.87e-11
	PP	-0.5	20	3.93e-11
	HA	[-0.5, 1/3]	7	4.43e-13

4. Conclusions

We presented a class of Hybrid Algorithm methods for finding simple zeros of nonlinear equations. In this paper, an algorithm Hybrid Algorithm is developed for computational purposes. The algorithm is tested on a number of numerical examples and the results obtain up to the desire accuracy are compared with the other methods. It is observed that our method takes a smaller number of iterations and more effective in comparison with these methods.

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6. CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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